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# Multiple imputation of systematically missing predictors in an individual participant data meta-analysis

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# Prediction models

Aim to predict...

- presence of a certain outcome (**diagnosis**)
- future occurrence of a certain outcome (**prognosis**)

Are based on...

- Individual characteristics
- Signs and symptoms
- More invasive or costly measures (e.g. imaging)

Are developed from...

- A set with individual participant data (IPD)
- Increasingly: multiple individual participant datasets  
Individual participant data meta-analysis (**IPD-MA**)



# IPD meta-analysis

## Between-study heterogeneity

- Differences in outcome prevalence/incidence
- Differences in predictor-outcome associations
- Should be avoided/mitigated in prediction models!!
- Missing data: impute datasets separately
- Problematic when some predictors are not measured in each individual dataset
  - Exclusion of entire studies or missing predictors
  - Use of imputation strategies ignoring heterogeneity

**Imputation strategies are needed to account for systematically missing data in an IPD-MA**



# Imputation of continuous systematically missing predictors

Previously, *Resche-Rigon* et al. developed a multiple imputation approach that<sup>1</sup>:

- Is based on MICE
- Imputes systematically missing continuous predictors
- Adopts linear mixed effect model with random intercept term and slopes
- Relies on standard error around estimated between-study covariance parameters

**Although promising, this approach is problematic for non-continuous predictors.**

<sup>1</sup> Resche-Rigon M et al. Multiple imputation for handling systematically missing confounders in meta-analysis of individual participant data. *Stat Med.* 2013 Dec 10;32(28):4890-905.



# Imputation of systematically missing predictors

- Standard errors of between-study covariance parameters are unreliable:
  - Likelihood of non-linear mixed effects models often lack a closed-form expression -> second-order derivatives become unreliable
  - Standard errors tend to be heavily skewed (even if log-transformed)
- Standard errors of between-study covariance parameters are not always reported (e.g. lme4)



# Imputation of non-continuous systematically missing predictors

- MICE procedure (assuming MAR)
- Generalized linear mixed effect model with
  - Fixed effects parameters ( $\boldsymbol{\gamma}$ )
  - Between-study covariance parameters ( $\boldsymbol{\Psi}$ )
  - Dispersion parameter(s) ( $\boldsymbol{\sigma}^2$ )  
(only for imputation of continuous predictors)
- Diffuse prior distributions for  $\boldsymbol{\gamma}$
- Prior distribution of  $\boldsymbol{\sigma}^2$  with density proportional to  $\sigma^{-2}$
- Reference prior for  $\boldsymbol{\Psi}^{-1}$



# The imputation procedure

Let  $M$  = number of studies where  $x$  is observed

1. Use MLE to estimate  $\boldsymbol{\gamma}$ ,  $\boldsymbol{\psi}$  and  $\boldsymbol{\sigma}^2$  in studies where  $x$  is observed
2. Draw  $\boldsymbol{\gamma}^*$  from  $\text{MVN}(\boldsymbol{\gamma}, \text{var}(\boldsymbol{\gamma}))$
3. Obtain random effects  $\mathbf{b}$  and calculate  $\Lambda = \text{sum}(\mathbf{b}^* \mathbf{b}^T)$
4. Draw  $\boldsymbol{\psi}^{*-1}$  from a Wishart distribution with  $\text{df}=M$  and scale matrix  $\Lambda^{-1}$
5. For studies where  $x$  is missing: draw  $\mathbf{b}^*$  from  $\text{MVN}(0, \boldsymbol{\psi}^*)$
6. For binary  $x$ : draw  $x^*$  using  $\text{logit}^{-1}(z\boldsymbol{\gamma}^* + z\mathbf{b}^*)$
7. For continuous  $x$ :  
draw  $\boldsymbol{\sigma}^{*2}$  using  $\sigma^2$  (based on  $X^2$  distribution)  
draw  $x^* = z\boldsymbol{\gamma}^* + z\mathbf{b}^* + \varepsilon^*$  where  $\varepsilon^* \sim N(0, \boldsymbol{\sigma}^{*2})$



# Empirical example

Diagnosis of deep vein thrombosis (DVT) I patients with a suspected DVT

- IPD meta-analysis of 13 studies (N=10,002)
- 11 predictors measured in all studies
- 4 (binary) predictors systematically missing
  - Results D-dimer test (*ddimmd*)
  - Family history of thrombophilia (*coag*)
  - Leg trauma presence (*notraum*)
  - Use of oral contraceptives (*oachst*)
- Estimation of coefficients *Oudega* model (8 predictors + intercept term)





# Methods

- Complete case analysis (**CCA**)  
exclude studies with missing predictor
- Traditional multiple imputation (**TMI**)  
imputation model ignoring between-study heterogeneity
- Multilevel multiple imputation (**MLMI**)  
imputation model accounting for between-study heterogeneity



# Empirical example results

Method		CCA	TMI	MLMI
<b>(intercept)</b>	$\beta$	-4.96	-5.00	-4.42
	SE( $\beta$ )	0.24	0.21	0.28
	$\tau$	0.29	0.46	0.77
(...)				
<b>ddimd</b>	$\beta$	2.68	2.69	2.06
	SE( $\beta$ )	0.18	0.15	0.34
	$\tau$	0.17	0.26	1.07
<b>notraum</b>	$\beta$	0.53	0.54	0.40
	SE( $\beta$ )	0.12	0.11	0.13
	$\tau$	0.00	0.03	0.18

CCA = complete case analysis

TMI = traditional multiple imputation

MLMI = multilevel multiple imputation



# Empirical example results

- Results CCA
  - Low degree of between-study heterogeneity
  - Solely based on Dutch studies
  - Poor transportability: MCAR not plausible (remaining studies are from different countries)
- Results TMI
  - Lowest standard errors
  - Medium levels of between-study heterogeneity
- Results MLMI
  - Largest standard errors
  - Largest degree of between-study heterogeneity



# Simulation study

- Based on DVT case study, but using 2 predictors that were measured in all studies
- Introduction of systematically missing predictors according to MCAR

## **Results** (not shown)

- Fixed effect estimates similar for all methods
  - Problematic coverage for TMI and CCA
- Substantial differences for between-study heterogeneity
  - Downward bias for CCA and TMI
  - MLMI sometimes yield extreme estimates when few studies were available



# Discussion

- CCA
  - Underestimates actual degree of heterogeneity
  - Problematic when MCAR not justified
  - Problematic when multiple predictors are missing, and almost all studies need to be excluded
- TMI
  - Underestimates actual degree of heterogeneity
- MLMI
  - Optimal coverage (predictor effects)
  - Lowest bias (between-study heterogeneity)
  - Possible issues: convergence & model complexity



# Discussion

CCA and TMI problematic during

- model development
  - Cannot properly identify homogeneous predictors
  - Detrimental selection of important predictors
- Model validation
  - Mask between-study heterogeneity, and therefore...
  - show overoptimistic model performance

**MLMI recommended to avoid bias in heterogeneity parameters and improve insight into potential model generalizability**

